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High-Order Mathematical Thinking and the Five-Units Mathematics Exam

Abstract based on a study by Boaz Silberman and Eli Kleinberger

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The international PISA assessments have been conducted since 2000, and place emphasis on the use of mathematical models to contend with problems from real-life contexts. To that end, students must be able to understand a problem, articulate it in mathematical language, and to apply and interpret the solution to which they arrived. As a result, the curriculum in Israel has been giving this approach more attention in recent years. The matriculation examinations have cautiously begun to include these types of tasks, mainly for the lower three and four-unit study tracks, and to a certain degree for the advanced five-unit level, as well.

In parallel, as of 2007, the Israeli Ministry of Education decided to bring learning to higher thinking and deeper comprehension (the "pedagogical horizon" policy). As part of this policy, the five-unit matriculation examination in mathematics was revised, and began to include more tasks at a high level of thinking, switching between representations, and complex mathematical abstraction. There are those who believe that this change was one of the causes for the decline in subsequent years in the number of students who successfully completed the matriculation examination at the five-unit level.

In order to confront this decline while keeping to the high standards, a broad national effort in recent years had succeeded to change course and to double the number of five-unit graduates. This process was achieved very quickly, at a fast pace and without changing the level of the examination. The question which now arises is, are the new students in the five units track less successful in the matriculation exam, and if so why is it happening? The hypothesis of this study is that if such a gap does exist, it would be characterized by differences in the levels of abstract thinking, and in levels of application skills.

Main findings

1. Mapping and analysis of the matriculation examinations of 2014-2019 found that only 15% of the questions in the five-unit mathematics matriculation examination were questions on the level of abstract thinking (according to the Ministry of Education's definition) and high order applied thinking (according to the PISA conceptual framework) – levels 4-5. The PISA conceptual framework includes a "super-level" (6) which is not included among the matriculation examination questions at all.

2. The data shows that the level of thinking has little influence on achievements due to the few questions at the high order thinking level and the low weight they contribute to the score. The single factor which predicts gaps in achievement is the topic of the question. The gaps are between "easy" topics (word problems, series, probabilities, trigonometry and analytic geometry) and "hard" topics (geometry, complex numbers and infinitesimal calculus).

3. For "easy" topics, the relationship between the level of thinking and achievements is discernable. A high level of thinking on an easy topic is a problematic threshold for struggling students. In contrast, for "hard" topics, the level of thinking on the question has no significant influence on the gap in achievements between the students. It appears that the main reason for this is that the topic itself is what creates the difficulty.

Frequency of items (sections and sub-sections) at each level of thinking/proficiency